USN

15MAT21

Second Semester B.E. Degree Examination, Feb./Mar. 2022 **Engineering Mathematics - II**

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

1 a. Solve
$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$$
 (05 Marks)

b. Solve
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 1 + 3x + x^2$$
 (05 Marks)

c. Solve
$$(D^2 + 1)y = 2 \cos x$$
 using method of undetermined coefficients. (06 Marks)

2 a. Solve
$$(D^2 + 1)y = \sin x \sin 2x$$
 (05 Marks)

a. Solve
$$(D^2 + 1)y = \sin x \sin 2x$$

b. Solve $(D + 2)(D - 1)^2 y = e^{-2x} + 2 \sinh x$ (05 Marks)

c. Solve
$$(D^2 + 1)y = \tan x$$
 using the method of variation of parameters. (06 Marks)

Module-2

3 a. Solve
$$x^2 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 4y = (1 + x)^2$$
 (05 Marks)

b. Solve
$$\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$$
 (05 Marks)

c. Solve
$$y - 2px = tan^{-1} (xp^2)$$
. (06 Marks)

4 a. Solve
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$$
 (05 Marks)

b. Solve
$$y = 2px + y^2p^3$$
 (05 Marks)

c. Solve
$$(px-y)$$
 $(py+x) = a^2p$ (by choosing $x^2 = u$ and $y^2 = v$) by reducing into Clairaut's form. (06 Marks)

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary constants from the relation
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
 (05 Marks)

b. Solve
$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$$
, given that $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ when y is odd

multiple of
$$\frac{\pi}{2}$$
. (05 Marks)

c. Derive one dimensional heat equation as
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial t^2}$$
 (06 Marks)

OR

- Form the partial differential equation by eliminating the arbitrary function given by $f(x + y + z, x^2 + y^2 + z^2) = 0$. (05 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ (05 Marks)
 - c. Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using method of separating of variables. (06 Marks)

- $\frac{\text{Module-4}}{\text{Evaluate}}$ Evaluate $\int_{0}^{4a} \int_{0}^{2\sqrt{ax}} dy dx$ by changing the order of integration. (05 Marks)
 - Find by triple integration the volume of the sphere $x^2 + y^2 + z^2 = r^2$ with a as the radius. (05 Marks)
 - c. Evaluate $\int_{0}^{2} \sqrt{\sin \theta} d\theta \times \int_{0}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}}$ using Beta and Gama functions. (06 Marks)

- OR $\int_{-\infty}^{\sqrt{1-x^2}} \int_{-\infty}^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-y^2-x^2}} dz dy dx$. (05 Marks)
 - Find the area lying between the parabola $y = 4x x^2$ and the line y = x. (05 Marks)
 - Evaluate $\int_{0}^{1} \frac{dx}{\sqrt{1+x^4}}$ using Beta and Gamma functions. (06 Marks)

- Find: $L\{\sin^3 2t + t^2e^{2t}\}$ (05 Marks)

Find the Laplace transform of the periodic function given by
$$f(t) = \begin{cases} 1, & 0 < t < \frac{a}{2} \\ -1, & \frac{a}{2} < t < a \end{cases} \text{ where } f(t+a) = f(t)$$
(05 Marks)

c. Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ using convolution theorem (06 Marks)

10 a. Find the Laplace transform of the function using unit step function given that

$$f(t) = \begin{cases} 2+t, & 0 < t < 1 \\ e^t, & t > 1 \end{cases}$$
 (05 Marks)

- $f(t) = \begin{cases} 2+t, & 0 < t < 1 \\ e^t, & t > 1 \end{cases}$ b. Find $L^{-1} \left\{ \frac{5s+3}{(s-1)(s^2+2s+5)} \right\}$ (05 Marks)
- c. Solve $\frac{d^2y}{dt^2} 2\frac{dy}{dt} + y = e^t$, y(0) = 2, y'(0) = -1 using Laplace transform. (06 Marks)